Int J Theor Phys (2009) 48: 1170–1176
DOI 10.1007/s10773-008-9890-z

Causality and Bell-CHSH Inequalities Violations in Entanglement Swapping Schemes

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Received: 26 August 2008 / Accepted: 29 October 2008 / Published online: 6 November 2008

Abstract An ample amount of evidence supporting the violation of locality has been presented in quantum theory. If such an instantaneous influencing is possible, it is worth considering the possibility of a causality violation in quantum theory, i.e., a future event influencing the past. Motivated by the delayed-choice gedanken experiment, we provide two protocols of entanglement swapping that are subtle in involving causality conditions. In particular, we present protocols in which locality constraints are identical to causality conditions and closely examine Bell-inequalities violation based on these protocols. These protocols will provide a clear picture of how quantum theory still preserves causality while locality is violated. We also discuss a potential threat to the entanglement-based key distribution schemes.

Keywords Bell inequalities · Locality · Causality

1 Introduction

Bell's theorem [1] and subsequent experimental verification [2] confirmed that the locality condition [3] is not preserved in quantum mechanics (see [4] for a review). That is, outcomes predicted by quantum theory cannot be reproduced locally. A large number of theoretical [5–7] and experimental [8–10] results have followed supporting this result. Since quantum theory violates the locality constraints, it seems reasonable to consider the possibility that quantum mechanics violates not only locality, but also causality. Wheeler proposed [11, 12] a scheme in which the path of a particle seems to depend on the later choice of detectors' positions, thereby implying that the past event may depend on the future event. This proposal became known as Wheeler's delayed-choice gedanken experiment and was also realized experimentally in [13].

In recent years, a number of applications of entanglement have been found, including cryptography [14], teleportation [15], and communication [16]. However, it is generally desired to create a long-distance entanglement in order to use these applications. One of the

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main advantages of entanglement-swapping schemes [17] is that they are able to create a long-distance entanglement using many short-range ones [18, 19]. Entanglement-swapping schemes are as follows: for qubits 1 and 2, 3 and 4 are maximally entangled. When a measurement is made with Bell basis on qubits 2 and 3, it creates a maximally entangled state between 1 and 4, i.e., the initial entanglement between 1 and 2, 3 and 4 are now swapped into 2 and 3, 1 and 4. Entanglement swapping has also been found useful in creating a quantum bus in computer architecture [20]. A few years ago, Peres showed a case where an entanglement-swapping scheme was considered, but with a little twist [21], i.e., a delayed-choice experiment using entanglement swapping. In the proposal, Peres shows that two particles that have never interacted before exhibit correlation depending on the measurement performed after the two particles are already measured. Later, Bruckner et al. [22] showed that the two particles not only exhibit entanglement, but that the degree of entanglement can also be post-created.

In this paper, we will provide two gedanken experiments involving entanglement swapping in which causality conditions appear to be violated. In particular, we will present gedanken experiments in which causality conditions are identical to the locality constraints. We then use Clauser-Horne-Shimony-Holt (CHSH) inequalities [5] and show that they appear to violate the inequalities; therefore, the locality conditions and the causality conditions are violated. We will present the first protocol that appears to violate causality conditions, then we will present a similar protocol that appears to violate causality conditions as well as locality conditions. However, we will show that the same result can be obtained using product states rather than entangled states, and we will provide an explanation as to why two protocols seem to violate causality. We will also discuss the possibility of simulating entanglement with product states when entanglement-swapping scheme is involved.

2 The First Protocol

Let us first introduce a notation. We define the usual Bell states as follows, $|\phi^{\pm}\rangle_{ij} \equiv \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{ij}$ and $|\psi^{\pm}\rangle_{ij} \equiv \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{ij}$. In our setup, two Bell states $|\phi^{+}\rangle_{12}$ and $|\phi^{+}\rangle_{34}$ are created at two separate regions, as shown in Fig. 1, and qubit 1 is carried to Alice, qubit 4 to Bob, and qubits 2 and 3 are delivered to Victor. We assume that Alice, Bob and Victor are all separated from each other in far distances. We now would like to introduce

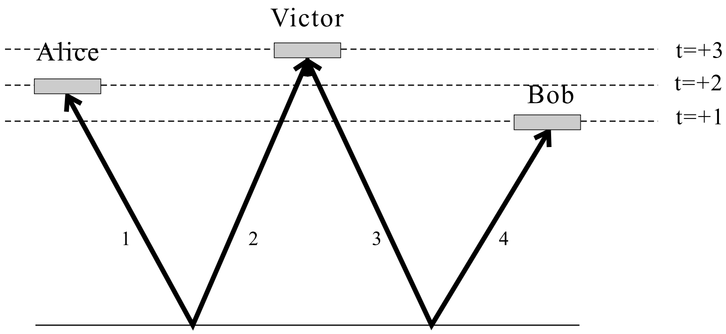

Fig. 1 The *horizontal coordinate* indicates a distance x and the *vertical coordinate* is for time t where time flows from *bottom* to *top*. Initially, two Bell states $|\phi^{+}\rangle_{12}$ and $|\phi^{+}\rangle_{34}$ are created at two distant regions. Then, qubit 1 is carried to Alice and measured at $t = +2$, qubit 4 to Bob and measured at $t = +1$, and qubits 2 and 3 are delivered to Victor and measured at $t = +3$

a notation to indicate the timing of measurements performed on qubits 1, 4, and 2 and 3 by Alice, Bob, and Victor, respectively. With $t = +1, +2, +3$, it is assumed that an event at $t = +1$ takes place before events at $t = +2, +3$, and an event at $t = +2$ takes place before an event at $t = +3$ and so on. We also assume that the time differences among $t = +1, +2, +3$ are much smaller compare to the distance between Alice, Bob, and Victor such that the events taken place at $t = +1, +2, +3$ cannot influence each other in a local way, i.e. at the speed of light at best. Now suppose qubit 4 is measured first by Bob at $t = +1$. Then at $t = +2$, qubit 1 is measured by Alice with the choice of basis between two distinct observables A_1 and A_2 (at this stage, we do not need to worry about what A_1 and A_2 are). Then at $t = +3$, Victor measures qubits 2 and 3 with his choice of basis between two different observables V_1 and V_2.

If we assume the choice of observables A_1 and A_2 is made right before the measurement of qubit 1, therefore after qubit 4 is measured at $t = +1$, then causality imposes that the outcome of qubit 4 cannot depend on A_i $(i = 1, 2)$. Note that, in this setting, even with an instantaneous influencing, it is not possible for the outcome of qubit 4 to be dependable on the Alice's choice of observables. As can be seen in Fig. 1, it would require a signal going back in time. If we denote the outcome of qubit 4 as $\mathcal{O}_4$, then causality implies (**c1**) $\mathcal{O}_4 \neq \mathcal{O}_4(A_i)$. Similarly if we assume the Victor's choice between V_1 and V_2 is made right before making a measurement on qubits 2 and 3 at $t = +3$, therefore after $t = +2$, causality imposes the second condition that the outcome of qubit 1 cannot depend on the choices of V_1 and V_2, or (**c2**) $\mathcal{O}_1 \neq \mathcal{O}_1(V_i)$ where $i = 1, 2$. Since the measurement on qubit 4 was made before the choice of Alice, the outcome of qubit 4 certainly cannot be dependent on V_i, i.e. (**c3**) $\mathcal{O}_4 \neq \mathcal{O}_4(V_i)$. Therefore in the setup shown in Fig. 1, causality implies all three conditions, (**c1–c3**), to be satisfied.

Next, we wish to discuss locality conditions for the setup we are considering as shown in Fig. 1. We assumed that the time differences among $t = +1, +2, +3$ are much smaller than the time it takes to signal between Alice, Bob, Victor in a local way (i.e. by the speed of light at best). Then under this setting, and with a locality assumption, we could draw the following conditions, i.e. (L1) the outcome of qubit 4 is independent of Alice's choice of observables A_1 and A_2, (L2) the outcome of qubit 1 is independent of V_1 and V_2, and (L3) the outcome of qubit 4 cannot depend on the Victor's choice between V_1 and V_2. The locality conditions (L1) and (L2) are the same as in the usual two particle entanglement for CHSH type inequalities [5] except that the Victor's choice between V_1 and V_2 replaced Bob's choice of observables. The condition (L3) is an extra locality condition added to the usual locality assumptions. Let us assume that $E(A_i, V_j)$ indicate the average value of outcomes for qubit 1 and 4 when the observables A_i and V_j are chosen. Based on the locality assumptions (L1), (L2), and (L3), we could obtain the following inequality condition for local models [5], $|E(A_1, V_1) + E(A_2, V_1) + E(A_1, V_2) - E(A_2, V_2)| \leq +2$ and $|E(A_1, V_1) + E(A_2, V_1) - E(A_1, V_2) + E(A_2, V_2)| \leq +2$ where, $E(A_i, V_j) = \int \mathcal{O}_1(A_i, \lambda)\mathcal{O}_4(\lambda)\rho(\lambda)d\lambda$ $(i, j = 1, 2)$ with λ as hidden variables, and $\rho(\lambda)$ as a probability density function. Note that the average values $E(A_i, V_j)$ are the same as in CHSH proof (where V_j are replace by B_j) except that we have an extra constraint, (L3), therefore $\mathcal{O}_4(\lambda)$ in $E(A_i, V_j)$ has no dependence on V_j.

We now wish to consider the outcome predicted by quantum mechanics. In order to do so, let us first introduce another notation, $|\chi^\pm\rangle_{ij} \equiv \frac{1}{\sqrt{2}}(|0+\rangle \pm |1-\rangle)_{ij}$, $|\eta^\pm\rangle_{ij} \equiv \frac{1}{\sqrt{2}}(|0-\rangle \pm |1+\rangle)_{ij}$ where $|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. With the newly defined states of $|\chi^\pm\rangle$ and $|\eta^\pm\rangle$, we can obtain the following relations, $|\phi^+\rangle_{12} \otimes |\phi^+\rangle_{34} = \frac{1}{2}(|\phi^+\rangle_{23} \otimes |\phi^+\rangle_{14} + |\phi^-\rangle_{23} \otimes |\phi^-\rangle_{14} + |\psi^+\rangle_{23} \otimes |\psi^+\rangle_{14} + |\psi^-\rangle_{23} \otimes |\psi^-\rangle_{14}) = \frac{1}{2}(|\chi^+\rangle_{23} \otimes |\chi^+\rangle_{14} + |\chi^-\rangle_{23} \otimes |\chi^-\rangle_{14} + |\eta^+\rangle_{23} \otimes |\eta^+\rangle_{14} + |\eta^-\rangle_{23} \otimes |\eta^-\rangle_{14})$. It represents usual entanglement swapping outcomes with Bell states. It can be seen that the newly defined states $|\chi^{\pm}\rangle$ and $|\eta^{\pm}\rangle$ also exhibit a similar entanglement swapping as shown in Bell states. Since $|\chi^{\pm}\rangle$ and $|\eta^{\pm}\rangle$ form an orthonormal basis for a two-qubit measurement, we let Victor's choice of observable V_1 correspond to the basis $\{|\phi^{\pm}\rangle_{23}, |\psi^{\pm}\rangle_{23}\}$ and V_2 to be $\{|\chi^{\pm}\rangle_{23}, |\eta^{\pm}\rangle_{23}\}$. Let us also define the observable choices for Alice as $A_1 = \frac{1}{\sqrt{2}}\sigma_x + \frac{1}{\sqrt{2}}\sigma_z$ and $A_2 = -\frac{1}{\sqrt{2}}\sigma_x + \frac{1}{\sqrt{2}}\sigma_z$ where σ_x and σ_z are Pauli matrices, $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$ and $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$. Bob will perform measurements on qubit 4 with only one observable, σ_z. Following Peres' proposal [21], after many runs of experiment in the setup of Fig. 1, we categorize the outcomes of qubit 1 and 4 into four sets. That is, the first set will be when Victor obtains $|\phi^+\rangle_{23}$ or $|\chi^+\rangle_{23}$, which we denote as $\{\phi^+_{23}, \chi^+_{23}\}$. The other three sets are $\{\phi^-_{23}, \chi^-_{23}\}$, $\{\psi^+_{23}, \eta^+_{23}\}$, and $\{\psi^-_{23}, \eta^-_{23}\}$. We now wish to consider what quantum theory predicts, after many runs of the experiment, about the outcomes of 1 and 4 will be for each subset. If we assume $\mathcal{E}(A_i, V_j)$ to be the average value of qubit 1 and 4 with A_i and V_j, after many runs of the experiment, quantum theory predicts the following result for the first set $\{\phi^+_{23}, \chi^+_{23}\}$, $\mathcal{E}(A_1, V_1) + \mathcal{E}(A_2, V_1) + \mathcal{E}(A_1, V_2) - \mathcal{E}(A_2, V_2) = +2\sqrt{2}$. This violates inequality condition derived based on the locality assumption (L1), (L2), and (L3). That is, the average values predicted from quantum theory cannot be reproduced from any theory with the locality assumptions (L1–L3).

For the other three sets, quantum theory predicts similar results that violate inequality conditions. For a set of $\{\phi^-_{23}, \chi^-_{23}\}$, we obtain the following relation from the prediction of quantum states, $\mathcal{E}(A_1, V_1) + \mathcal{E}(A_2, V_1) - \mathcal{E}(A_1, V_2) + \mathcal{E}(A_2, V_2) = +2\sqrt{2}$. For the remaining two sets $\{\psi^+_{23}, \eta^+_{23}\}$ and $\{\psi^-_{23}, \eta^-_{23}\}$, quantum prediction yields $\mathcal{E}(A_1, V_1) + \mathcal{E}(A_2, V_1) - \mathcal{E}(A_1, V_2) + \mathcal{E}(A_2, V_2) = -2\sqrt{2}$ and $\mathcal{E}(A_1, V_1) + \mathcal{E}(A_2, V_1) + \mathcal{E}(A_1, V_2) - \mathcal{E}(A_2, V_2) = -2\sqrt{2}$. Therefore, all four sets violate the inequalities. The violations of inequality imply the violation of locality conditions, i.e. (L1), (L2), and (L3). Since the locality conditions (L1–L3) were identical to the causality conditions **(c1–c3)**, it appears that the violation of inequality leads to the violation of causality conditions. Therefore it is not immediately clear why the conclusion of this protocol cannot be true. We wish to provide a second protocol in which why causality cannot be violated can be shown more vividly.

3 The Second Protocol

Two Bell states $|\phi^+\rangle_{12}$ and $|\phi^+\rangle_{34}$ are created from a single source, as shown in Fig. 2, and qubit 1 is carried to Alice, qubit 4 to Bob, and qubits 2 and 3 are delivered to Victor. We assume that Alice, Bob and Victor are all separated from each other in far distances. Similar to the first protocol, with $t = +1, +2$, it is assumed that an event at $t = +1$ takes place before events at $t = +2$. It is also assumed that the time difference among $t = +1, +2$ is much smaller compare to the distances between Alice, Bob, and Victor such that the events taken place at $t = +1, +2$ cannot influence each other in a local way, i.e. at the speed of light at best. Now suppose qubits 1 and 4 are measured first by Alice and Bob, respectively, at $t = +1$. Then at $t = +2$, Victor measures qubits 2 and 3 with his choice of basis among four different observables V_{ij} where $i, j = 1, 2$.

If we assume the Victor's choice among V_{ij}'s is made right before making a measurement on qubits 2 and 3 at $t = +2$, therefore after $t = +1$, causality imposes that the outcomes of qubit 1 and 4 cannot depend on the choice of V_{ij}. Note that, in this setting, even with an instantaneous influencing, it is not possible for the outcome of qubit 1 and 4 to be dependable on the Victor's choice of observables. We will assume that Alice and Bob will measure a single observable for qubits 1 and 4. If we denote the outcome of qubit 1 and 4 as $\mathcal{O}_1$ and

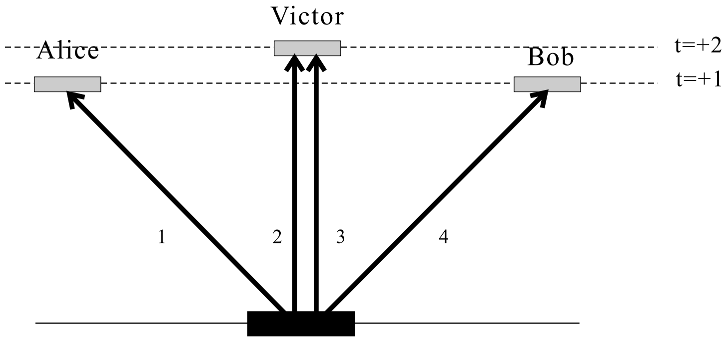

Fig. 2 The *horizontal coordinate* indicates a distance x and the *vertical coordinate* is for time t where time flows from *bottom* to *top*. Initially, two Bell states $|\phi^+\rangle_{12}$ and $|\phi^+\rangle_{34}$ are created from a single resource. Then, qubit 1 is carried to Alice and measured at $t=+1$, qubit 4 to Bob and measured at $t=+1$, and qubits 2 and 3 are delivered to Victor and measured at $t=+2$

$\mathcal{O}_4$, respectively, in the setup shown in Fig. 2, causality implies the following conditions, (**c′1**) $\mathcal{O}_1 \neq \mathcal{O}_1(V_{ij})$ and (**c′2**) $\mathcal{O}_4 \neq \mathcal{O}_4(V_{ij})$ to be satisfied.

Next, we discuss locality conditions for the setup we are considering as shown in Fig. 2. We assumed that the time difference between $t=+1,+2$ is much smaller than the time it takes to signal between Alice, Bob, Victor in a local way (i.e. by the speed of light at best). Then under this setting, and with a locality assumption, we could draw the following conditions, i.e. (L′1) the outcome of qubit 1 is independent of Victor's choice of observables V_{ij}, (L′2) the outcome of qubit 4 is independent of V_{ij}. The locality conditions (L′1) and (L′2) are the same as in the usual two particle entanglement for CHSH type inequalities [5] except that the Victor's choice V_{ij} replaced Alice and Bob's choice of observables. Let us assume that $E(V_{ij})$ indicate the average value of outcomes for qubit 1 and 4 for the observables V_{ij}. Based on the locality assumptions (L′1) and (L′2), we could obtain the following inequality condition for local models [5], $|E(V_{11})+E(V_{21})+E(V_{12})-E(V_{22})| \leq +2$ and $|E(V_{11})+E(V_{21})-E(V_{12})+E(V_{22})| \leq +2$ where $E(V_{ij}) = \int \mathcal{O}_1(\lambda)\mathcal{O}_4(\lambda)\rho(\lambda)d\lambda$ where $i,j=1,2$, λ as hidden variables, and $\rho(\lambda)$ as a probability density function. Note that the average values $E(V_{ij})$ are the same as in CHSH proof (where V_{ij} are replaced by A_i and B_j).

We now wish to consider the outcome predicted by quantum mechanics. In order to do so, let us first introduce another notation, $|+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, $|+'\rangle \equiv \alpha|0\rangle + \beta|1\rangle)$, $|-'\rangle \equiv \beta|0\rangle - \alpha|1\rangle)$, $|+''\rangle \equiv \gamma|0\rangle + \delta|1\rangle)$, $|-''\rangle \equiv \delta|0\rangle - \gamma|1\rangle)$ where $\alpha = \cos\frac{\pi}{8}, \beta = \sin\frac{\pi}{8}, \gamma = \cos\frac{-\pi}{8}$, and $\delta = \sin\frac{-\pi}{8}$. We also define, $|\Phi^\pm(1,1)\rangle \equiv \frac{1}{\sqrt{2}}(|+'\rangle|0\rangle \pm |-'\rangle|1\rangle)$, $|\Psi^\pm(1,1)\rangle \equiv \frac{1}{\sqrt{2}}(|+'\rangle|1\rangle \pm |-'\rangle|0\rangle)$, $|\Phi^\pm(1,2)\rangle \equiv \frac{1}{\sqrt{2}}(|+'\rangle|+\rangle \pm |-'\rangle|-\rangle)$, $|\Psi^\pm(1,2)\rangle \equiv \frac{1}{\sqrt{2}}(|+'\rangle|-\rangle \pm |-'\rangle|+\rangle)$, $|\Phi^\pm(2,1)\rangle \equiv \frac{1}{\sqrt{2}}(|+''\rangle|0\rangle \pm |-''\rangle|1\rangle)$, $|\Psi^\pm(2,1)\rangle \equiv \frac{1}{\sqrt{2}}(|+''\rangle|1\rangle \pm |-''\rangle|0\rangle)$, $|\Phi^\pm(2,2)\rangle \equiv \frac{1}{\sqrt{2}}(|+''\rangle|+\rangle \pm |-''\rangle|-\rangle)$, and $|\Psi^\pm(2,2)\rangle \equiv \frac{1}{\sqrt{2}}(|+''\rangle|-\rangle \pm |-''\rangle|+\rangle)$. With the newly defined states, we can obtain the following relations, $|\phi^+\rangle_{12} \otimes |\phi^+\rangle_{34} = \frac{1}{2}(|\Phi^+(i,j)\rangle_{23} \otimes |\Phi^+(i,j)\rangle_{14} + |\Phi^-(i,j)\rangle_{23} \otimes |\Phi^-(i,j)\rangle_{14} + |\Psi^+(i,j)\rangle_{23} \otimes |\Psi^+(i,j)\rangle_{14} + |\Psi^-(i,j)\rangle_{23} \otimes |\Psi^-(i,j)\rangle_{14})$ where $i,j=1,2$. This shows that the newly defined states $|\Phi^\pm(i,j)\rangle$ and $|\Psi^\pm(i,j)\rangle$ also exhibit a similar entanglement swapping as shown in Bell states. We now define Vector's choice of observables as follows, $V_{ij} \equiv \{|\Phi^\pm(i,j)\rangle_{23}, |\Psi^\pm(i,j)\rangle_{23}\}$ where each set for $i,j=1,2$ forms an orthonormal basis. Alice and Bob will perform measurements on qubit 1 and 4 with only one observable, σ_z.

Similar to the first protocol, after many runs of experiment in the setup of Fig. 2, we categorize the outcomes of qubit 1 and 4 into the following four sets: $\{|\Phi^+(i,j)\rangle_{23}\}$, $\{|\Phi^-(i,j)\rangle_{23}\}$, $\{|\Psi^+(i,j)\rangle_{23}\}$, $\{|\Psi^-(i,j)\rangle_{23}\}$ where $i, j = 1, 2$. We now wish to consider what quantum theory predicts, after many runs of the experiment, about the outcomes of 1 and 4 will be for each subset. If we assume $\mathcal{E}_{\Phi^+}(V_{ij})$ to be the average value of qubit 1 and 4 for the subset in $\{|\Phi^+(i,j)\rangle_{23}\}$ when Victor chooses to measure V_{ij}, quantum theory predicts, after many runs of the experiment, the following result, $\mathcal{E}_{\Phi^+}(AB|V_{11}) + \mathcal{E}_{\Phi^+}(AB|V_{21}) + \mathcal{E}_{\Phi^+}(AB|V_{12}) - \mathcal{E}_{\Phi^+}(AB|V_{22}) = +2\sqrt{2}$. This violates the inequality condition derived based on the locality assumption (L$'$1) and (L$'$2). That is, the average values predicted from quantum theory cannot be reproduced from any theory with the locality assumptions (L$'$1) and (L$'$2). For the other three sets, quantum theory predicts similar results that violate inequality conditions. For a set $|\Phi^-(i,j)\rangle$, we obtain the following relation from the prediction of quantum states, $\mathcal{E}_{\Phi^-}(AB|V_{11}) + \mathcal{E}_{\Phi^-}(AB|V_{21}) - \mathcal{E}_{\Phi^-}(AB|V_{12}) + \mathcal{E}_{\Phi^-}(AB|V_{22}) = +2\sqrt{2}$. For the remaining two sets, $|\Psi^+(i,j)\rangle$ and $|\Phi^-(i,j)\rangle$, quantum prediction yields $\mathcal{E}_{\Psi^+}(AB|V_{11}) + \mathcal{E}_{\Psi^+}(AB|V_{21}) + \mathcal{E}_{\Psi^+}(AB|V_{12}) - \mathcal{E}_{\Psi^+}(AB|V_{22}) = -2\sqrt{2}$ and $\mathcal{E}_{\Psi^-}(AB|V_{11}) + \mathcal{E}_{\Psi^-}(AB|V_{21}) - \mathcal{E}_{\Psi^-}(AB|V_{12}) + \mathcal{E}_{\Psi^-}(AB|V_{22}) = -2\sqrt{2}$. Therefore, all four sets violate the Bell-CHSH inequalities. The violations of inequality appear imply the violation of locality conditions, i.e. (L$'$1) and (L$'$2).

4 No Causality Violation

We considered a setting where locality conditions (L$'$1) and (L$'$2) are identical to causality conditions (**c$'$1**) and (**c$'$2**). Therefore, it seems that the quantum prediction violates the locality condition, and therefore the causality conditions. However, let us consider the following scheme. That is, we wish to re-consider the setup shown in Fig. 2, however, rather than two entangled Bell states $|\phi^+\rangle_{12}$ and $|\phi^+\rangle_{34}$, the source creates the following product states, with $P = 1/4$ for each, $|0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3 \otimes |0\rangle_4$, $|0\rangle_1 \otimes |0\rangle_2 \otimes |1\rangle_3 \otimes |1\rangle_4$, $|1\rangle_1 \otimes |1\rangle_2 \otimes |0\rangle_3 \otimes |0\rangle_4$, and $|1\rangle_1 \otimes |1\rangle_2 \otimes |1\rangle_3 \otimes |1\rangle_4$. As before, Alice and Bob will measure qubits 1 and 4, respectively, with one observable σ_z and Victor will choose between four bases V_{ij} (where $i, j = 1, 2$) to measure qubits 2 and 3. We then categorize the outcomes of qubit 1 and 4 depending on the measurement result by Victor for qubits 2 and 3 according to the four sets, $\{|\Phi^+(i,j)\rangle_{23}\}$, $\{|\Phi^-(i,j)\rangle_{23}\}$, $\{|\Psi^+(i,j)\rangle_{23}\}$, $\{|\Psi^-(i,j)\rangle_{23}\}$. For example, for a set for $|\Phi^+(i,j)\rangle$, we calculate the average value for qubits 1 and 4 when V_{11} is chosen. This yields the same value as $\mathcal{E}_{\Phi^+}(V_{11})$ in the relation from quantum prediction, $\mathcal{E}_{\Phi^+}(AB|V_{11}) + \mathcal{E}_{\Phi^+}(AB|V_{21}) + \mathcal{E}_{\Phi^+}(AB|V_{12}) - \mathcal{E}_{\Phi^+}(AB|V_{22}) = +2\sqrt{2}$. Similarly, the average can be obtained for V_{12}, V_{21}, V_{22} and it satisfies the outcome predicted in case of two entangled initial Bell states. Similarly, for each sets, $\{|\Phi^-(i,j)\rangle_{23}\}$, $\{|\Psi^+(i,j)\rangle_{23}\}$, $\{|\Psi^-(i,j)\rangle_{23}\}$, the outcomes of qubits 1 and 4 will be the same as in the relations from quantum predictions, i.e., the case with entangled states. Therefore, with product states, Bell inequalities are still violated. We therefore see that the violation of Bell-CHSH inequalities appear to be violated even with product states just as in the entangled Bell states. There is no violation of causality because it is only when Victor categorizes into four groups, the violation occurs. One can see that after Alice or Bob measures at $t = +1$, the state remaining for Victor at $t = +2$ are $|00\rangle_{23}$, $|01\rangle_{23}$, $|10\rangle_{23}$ and $|00\rangle_{23}$ with equal probabilities. Therefore, this can be simulated with product states as shown above. Similarly in the first protocol, when Alice and Bob measure qubits 1 and 4, the corresponding states for qubits 2 and 3 are prepared for Victor. Then Victor may obtain the outcomes on these states by performing a measurement with V_1 or V_2. Again, the violation of Bell-CHSH inequalities only occurs after the categorization into groups.

One of the main points of entanglement swapping is that after Bell measurement, the remaining states fall into four Bell states rather than one. If the outcome was just one Bell state with unit probability that violates inequalities, we could have had causality violation in the two protocols provided above. The situation is similar to the Deutsch and Hayden's Heisenberg picture formulation of teleportation [23] in which no information is transferred nonlocally, but contained in the two-bit classical information, i.e., the property of nonlocality is exhibited depending on the measurement outcomes of Bell measurement. This may also pose a potential threat to the security of the quantum key distribution schemes using entanglement and Bell-inequalities violation. Suppose Eve has taken over Victor's end, then it is possible to send Alice and Bob the product states. Then Eve could announce her measurement results and know exactly what Alice and Bob obtained, while Alice and Bob's results categorized according to Eve's announcement would still violate the inequalities.

Acknowledgements We wish to thank K. Matsumoto, W.J. Munro, and J. Bae for very helpful discussions.

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